

Empirical Modelling of a Rayleigh Fading Channel to Compute the Channel Capacity using SVD

Sourav Bhattacharyya
*Electronics and Telecommunication
 Engineering*
 Dr. B. C. Roy Polytechnic
 Durgapur, India
sourav.bhattacharyya@bcrec.ac.in

Karunamoy Chatterjee
*Electronics and Communication
 Engineering*
 Muzaffar Ahmed Mahavidyalaya
 Murshidabad, India
karuna_ds@rediffmail.com

Aritra Bhowmik
*Electronics and Communication
 Engineering*
 Dr. B. C. Roy Engineering College
 Durgapur, India
aritra.bhowmik@bcrec.ac.in

Abstract—This work presents the state-of-the-art of wireless communication system performance metrics through the reviewing of multi-path propagation model and Singular-Value-Decomposition (SVD) method to enhance the channel capacity in the 4th and 5th generation wireless communication systems. MIMO has shown rapid improvement in data rate by lowering the Signal-to-Noise ratio (SNR). In multi-path propagation, fading is a serious issue that increases the overall average Bit-Error Rate (BER). In this research, an improvement of data rate in terms of enhancing the channel capacity of the MIMO system is summarized by reducing the SNR using SVD. An Empirical model of the MIMO system is studied, and an IID Rayleigh-Fading MIMO system is modeled to compute the channel capacity.

Keywords—RD, IID, MIMO, SVD, BER

I. INTRODUCTION

The fifth-generation technology has received wide attention and a lot of research from global enterprises, research institutes, and universities to improve the spectrum utilization and as well as the channel capacity of the wireless communication systems. Therefore, the wireless communication system needs to meet the higher data transmission/reception rate and higher system capacity. Therefore, the communication system demands to utilize the bandwidth resource efficiently. Due to the shortage of spectrum resources, it is important to improve the spectrum utilization of the system for future communication technologies [1]. MIMO technology was first proposed by Marconi in 1908. In MIMO, multiple antennas on both the sending and receiving ends are utilized to improve the capacity of the communication system, transmission rate of system data, and transmission reliability.

Since the first MIMO approach was patented in 1994 by T. Kailath and A. J. Paulraj, MIMO has been a well-researched topic in wireless communication and is the 100th recipient of the Faraday medal for “the invention, advancement, and commercialization of MIMO Wireless” in 2023.

A. Basics of MIMO

MIMO was a paradigm shift in wireless transmission because it broke fundamental barriers to increasing data rates. Before MIMO, enhancing the link speed needed either more bandwidth and, or higher transmit power. But they suffer from their perspectives as both bandwidth and power were

insurmountable barriers. MIMO multiplied data rates many folds without the need for increased bandwidth or transmit power [2]. Parthasarathy says, ‘the transmit power advantage of MIMO is striking; a 4x4 MIMO link can offer a million times the power advantages of a standard 1x1 link. MIMO is today the core enabler of 4G and 5G mobile and Wi-Fi networks. The 6G standard currently being developed will also use MIMO and promise even higher data rates and efficient delivery of AI services.

B. Comparison Between SISO and MIMO System

In conventional SISO system, the data rate can be increased by either increasing the transmission bandwidth or transmit power. But both the techniques are not fruitful as the frequency spectrum is a valuable resource and in the other hand, we cannot provide large power to the antennas as it reduces the battery life.

Whereas, MIMO increases the spectrum efficiency without increasing the transmission power and bandwidth. There are two types of gain; rate gain and diversity gain are popularly used in MIMO. For parallel MIMO channel, there is at the most minimum {NR, NT} rate gain from that of a SISO system which is popularly known as multiplexing gain. Therefore, in the spatial multiplexing of the MIMO systems different data are sent through the parallel channels with the help of a serial to parallel converter to provide higher transmission rate. The maximum number of independent paths travelled by each signal can be at the maximum {NR, NT}. It is highly possible that not all the paths are highly faded. In this case same data is sent through all the multiple antennas at the transmitter [3-6]. If any of the path is completely down, the other paths will still be working then the receiver tries to make an efficient use of this technique to decode the data accordingly, which gives higher link reliability. In fact, there is trade-off popularly known as diversity-multiplexing trade-off between the two fundamental gains; rate gain and diversity gain.

C. Key Concept of Different Fading Issues

Even though wireless transmission has completely transformed the way of communication, a number of issues still exist. When there are variations in the signal strength, fading is a serious issue. The main offenders of log-normal shadowing, or gradual fading, are trees, buildings, and topographical features. As the name implies, slow fading causes gradual variations in signal strength over comparatively long periods of time. This kind of fading is more common outside when there are a lot of obstacles in the

way of the signal. Robust error correction and detection techniques are necessary to ensure reliable communication due to the features of slow fading, particularly in situations where the received signal power varies gradually over time. The effects of slow fading on wireless transmission are significant. In order to offset the signal loss caused by gradual fading, complex modulation techniques and signal amplification are frequently used in mitigation. Furthermore, adaptive antenna systems are utilized to dynamically modify the signal transmission in order to counteract slow fading and guarantee dependable connectivity throughout different types of terrain. In contrast to slow fading, fast fading is characterized by sudden variations in signal strength that typically occur over short time intervals. This phenomenon is sometimes attributed to multipath propagation, in which the supplied signal travels over multiple paths before reaching the receiver and causing both constructive and destructive interference.

Fast fading is a significant issue for wireless communication systems since it can result in errors and distortion due to sudden changes in signal strength. Adaptive modulation, coding techniques, and dynamic channel equalization are used to offset the fast signal fluctuations and decrease the impacts of rapid fading. Variations in the attenuation and delay of the transmitted signal cause frequency-selective fading, which is also referred to as frequency- or time-dispersive fading. Significant multipath propagation is a common context for this kind of fading, which causes frequency-dependent signal distortions. Wireless systems use methods like orthogonal frequency-division multiplexing (OFDM) and frequency-domain equalization to counteract the effects of frequency-selective fading. These approaches help to reduce distortions related to frequency and guarantee reliable communication across a range of frequency components. Inter-symbol interference and spectral distortion are brought on by frequency-selective fading, which makes it more challenging to reliably recover the supplied data [7-12].

One important way to lessen the impact of frequency-selective fading is to use equalization techniques to counteract the frequency-dependent channel defects. Adaptive equalizers minimize distortion and provide accurate signal recovery from frequency-selective fading. Furthermore, orthogonal frequency-division multiplexing (OFDM) modulation mitigates frequency-selective fading and efficiently lowers the effects of frequency-dependent channel variations by splitting the transmission into several narrowband subcarriers.

II. ANTENNA DIVERSITY

In Single-Input-Single-Output (SISO), Low Density Parity Check Code and Turbo Codes with iterative decoding algorithms are the capacity booster, whereas, in Single-Input-Multiple-Output (SIMO), receiver diversity techniques such as equal gain combining (EGC), selection combining (SC), and maximal-ratio-combining (MRC) are utilized to combat multi-path fading. MRC is optimal in terms of SNR but complex to implement in terms of the other combining scheme. For Multi-Input-Single-Output (MISO), the receiver diversity is not cost impressive. Instead, transmit diversity at the base station is better choice. Whereas, in MIMO system both the transmit and received diversity techniques are utilized rather in individual techniques [13-16].

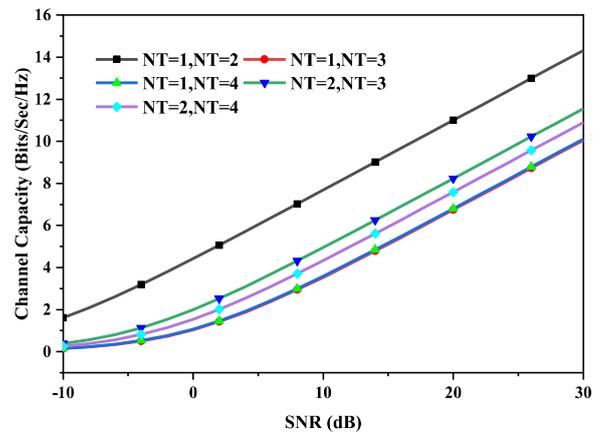
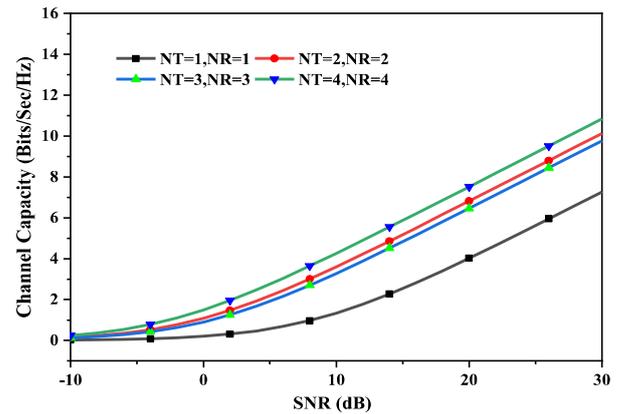


Fig. 1. Channel Capacity of Different Antenna Diversity

A. Different Diversity Scheme

There are three transmit diversity scheme have been employed based on the Channel-State-Information (CSI). When the CSI is available at both the transmitter and receiver, it is called closed loop MIMO system. In open loop MIMO, CSI is available at the receiver but not at the transmitter. But when the CSI is not available at both the transmitter and receiver, it is called blind MIMO system and should operate in non-coherent mode. Figure 1 shows that the average channel capacity of the MIMO antenna is 10 bits/Sec/Hz.

B. Diversity-Multiplexing Trade-off

The rate gain is associated with the data rate transmission. The rate gain

$$r = \lim_{SNR \rightarrow \infty} \frac{R(SNR)}{\log_2(SNR)}$$

The diversity gain is associated with the probability of error in detection. A transmission scheme is said to achieve diversity gain d , if the probability of error is $P_e(SNR)$, as function of SNR satisfies,

$$d = - \lim_{SNR \rightarrow \infty} \frac{\log_2\{P_e(SNR)\}}{\log_2(SNR)}$$

For a given rate gain r , the optimal diversity gain $d_{opt}(r)$, is the supreme diversity gain that can be accomplished by any MIMO system. If the fading block length is T such that,

$$T \geq N_T + N_R - 1$$

The optimal diversity gain can be calculated as,

$$d_{opt}(r) = (N_T - r)(N_R - r), 0 \leq r \leq \min(N_T - N_R)$$

III. MATHEMATICAL MODELLING OF MULTIPATH ENVIRONMENT

In a multipath environment, the wireless signal reaches the mobile receiver from the Base-Station (BS) through the different paths, some of the links use Line-of-Sight (LOS) paths and some reaches through different scattering components. As a results, the mobile receiver leads to superposition of those multiple signals, i.e., LOS and NLOS. As, in the multiple scenarios, there are L components, therefore considering the k th path which can be characterized by a delay of τ_k and attenuation of a_k . According to the principles, the received signal is superimposing of the multiple components at the receiver, therefore, the multipath response of the channel can be modelled as the sum of individual path response as dissipated in equation (1).

$$h(t) = \sum_{k=0}^{L-1} a_k \delta(t - \tau_k) \dots \dots \dots (1)$$

for the k th path, the received signal is denoted as

$$y_p(t) = \text{Re} \left\{ \left(\sum_{k=0}^{L-1} a_k S(t - \tau_k) e^{-j2\pi F_c \tau_k} \right) e^{j2\pi F_c t} \right\}, \dots \dots (2)$$

Where,

$$\sum_{k=0}^{L-1} a_k S(t - \tau_k) e^{-j2\pi F_c \tau_k} \text{ is received baseband signal}$$

and $e^{j2\pi F_c t}$ Carrier Component

Therefore, the received baseband signal becomes,

$$y(t) = \left(\sum_{k=0}^{L-1} a_k e^{-j2\pi F_c \tau_k} \right) S(t) \dots \dots \dots (3)$$

As,

$$h = \sum_{k=0}^{L-1} a_k e^{-j2\pi F_c \tau_k}$$

therefore, equation (3) can be rewrite as,

$$y(t) = h \times S(t) \dots \dots \dots (4)$$

As the channel coefficient (h) varies depending on the various channel attenuation factor a_k and delay τ_k , therefore, the channel coefficient is also called the Fading-Channel

Coefficient. This fading process causes the received power to vary, which is a key barrier of the wireless communication.

A. Fading Channel Distribution Model

From the fading channel coefficient,

$$h = \sum_{k=0}^{L-1} a_k e^{-j2\pi F_c \tau_k}$$

can further be rewritten as,

$$h = \sum_{k=0}^{L-1} a_k \cos(2\pi F_c \tau_k) - j \sum_{k=0}^{L-1} a_k \sin(2\pi F_c \tau_k) \dots \dots (5)$$

Where,

$$x = \sum_{k=0}^{L-1} a_k \cos(2\pi F_c \tau_k) \text{ and}$$

$$y = - \sum_{k=0}^{L-1} a_k \sin(2\pi F_c \tau_k)$$

Since, x and y are the sums of many random components involved in attenuation and delay, they are also random in nature. By the Central Limit Theorem (CLT), x and y can be assumed to be the Gaussian Distribution in nature and x and y are independent Gaussian-Random Variable, with zero mean and variance to $1/2$.

B. Characterization of Fading Channel Coefficient

The marginal distribution of the amplitude may also be called as Rayleigh Distribution and the channel coefficient h is also termed as Rayleigh Fading Channel. Then the amplitude of the marginal Rayleigh Distribution of the equation may also be written as follows,

$$F_A(a) = \int_{-\pi}^{\pi} F_{A,\varphi}(a, \varphi) d\varphi = 2ae^{-a^2}$$

And,

$$F_{A,\varphi}(a, \varphi) = \frac{a}{\pi} e^{-a^2}$$

The distribution of the phase can be written as,

$$F_\varphi(\varphi) = \int_0^\infty F_{A,\varphi}(a, \varphi) da = \frac{1}{2\pi} \text{ for } -\pi < \varphi \leq \pi$$

IV. ESTIMATION OF BIT ERROR RATE

There are many matrices to evaluate the performance of the wireless communication system. One of the most convenient matrices is the Bit Error Rate (BER) which is also called the Average Bit Error Rate which is often expressed by probability which lies between 0 and 0.5. The maximum possible BER is 0.5. In the Binary Phase Shift Keying (BPSK), the symbols 0 and 1 are presented as,

$$0 \rightarrow \sqrt{P} \text{ and } 1 \rightarrow -\sqrt{P}$$

Therefore, there is 180° phase-shifting between the symbols and the average power in this BPSK format is P. Under the consideration of noise in the channel, the received signal can be expressed as,

$$y = hx + n$$

Then the received power can be calculated as,

$$\text{Received Power} = |h|^2 P = a^2 P$$

So, the SNR due to fading,

$$\text{SNR}_F = \frac{a^2 P}{\sigma^2} = a^2 \text{SNR}$$

The BER of the BPSK modulation is written as,

$$\text{BER} = Q\sqrt{\text{SNR}_F} = Q\sqrt{a^2 \text{SNR}}$$

Whose solution becomes,

$$\text{Average BER} = \frac{1}{2} \left(1 - \sqrt{\frac{\text{SNR}}{2 + \text{SNR}}} \right)$$

This is the average BER of a Rayleigh Fading Channel.

V. ESTIMATION OF POWER PROFILE IN MULTIPATH SCENARIO

The power of the *i*th multipath component can be given as,

$$\varphi(t) = \sum_{i=0}^{L-1} |a_i|^2 \delta(t - \tau_i)$$

$$\text{if } |a_i|^2 = g_i$$

$$\varphi(t) = \sum_{i=0}^{L-1} g_i \delta(t - \tau_i)$$

there is a power *a_i* or the gain *g_i* which is arriving with a delay of *τ_i* which gives the power profile of the wireless channel. For L=4 receiving antennas of a multipath channel,

$$\varphi(t) = \sum_{i=0}^3 g_i \delta(t - \tau_i)$$

therefore, the gain *g₀*, *g₁*, *g₂*, and *g₃* are the gain at different delay *τ₀*, *τ₁*, *τ₂*, and *τ₃* respectively. So, the maximum delay spread may be given as,

$$\sigma_\tau^{Max} = \tau_3 - \tau_0$$

In general terms, the maximum delay spread is represented by,

$$\sigma_\tau^{Max} = \tau_{L-1} - \tau_0$$

This maximum delay spread (MDS) is another metric to characterize the delay spread of the wireless communication channel but this MDS is not very appropriate, as in the power profile there might be very small power contents at the far distance are negligible and called the spurious components. In such scenarios, instead of MDS an alternative metric called RMS delay spread becomes very useful. The fraction of power at the *i*th path,

$$b_i = \frac{g_i}{\sum_{j=0}^{L-1} g_j}$$

Then the average delay,

$$\bar{\tau} = b_0 \tau_0 + b_1 \tau_1 + \dots + b_{L-1} \tau_{L-1}$$

So,

$$\bar{\tau} = \sum_{i=0}^{L-1} b_i \tau_i = \sum_{i=0}^{L-1} \frac{g_i \tau_i}{\sum_{j=0}^{L-1} g_j}$$

Therefore, the average delay,

$$\bar{\tau} = \frac{\sum_{i=0}^{L-1} g_i \tau_i}{\sum_{i=0}^{L-1} g_i}$$

So, the RMS delay spread is given by the deviation of the multipath power profile,

$$\sigma_\tau^2 = b_0 (\tau_0 - \bar{\tau})^2 + b_1 (\tau_1 - \bar{\tau})^2 + \dots + b_{L-1} (\tau_{L-1} - \bar{\tau})^2$$

Then,

$$\sigma_\tau = \left(\sum_{i=0}^{L-1} b_i (\tau_i - \bar{\tau})^2 \right)^{\frac{1}{2}}$$

Therefore, the RMS delay spread, in terms of gain,

$$\sigma_\tau = \left(\frac{\sum_{i=0}^{L-1} g_i (\tau_i - \bar{\tau})^2}{\sum_{i=0}^{L-1} g_i} \right)^{\frac{1}{2}}$$

$$\text{And as, } g_i = |a_i|^2$$

Therefore, the RMS delay spread of the multipath delay profile,

$$\sigma_\tau = \left(\frac{\sum_{i=0}^{L-1} |a_i|^2 (\tau_i - \bar{\tau})^2}{\sum_{i=0}^{L-1} |a_i|^2} \right)^{\frac{1}{2}}$$

TABLE I. AVERAGE DELAY SPREAD AND RMS DELAY SPREAD

The table below shows the different values of *τ_i* and *g_i* for L=4 multipath components.

<i>τ_i</i> (μs)	<i>g_i</i>	dB Gain	<i>a</i> = √ <i>g</i>
0	0.01	-20	0.1

1	0.1	-10	0.316
2	1	0	1
3	0.1	-10	0.316

Using the equation (),

$$\bar{\tau} = \frac{0.01 \times 0 + 0.1 \times 1 + 1 \times 2 + 0.1 \times 3}{0.01 + 0.1 + 1 + 0.1} = 1.98 \mu\text{s}$$

Now, the RMS delay spread using the equation (),

$$\sigma_{\tau} = \left(\frac{\sum_{i=0}^{L-1} |a_i|^2 (\tau_i - \bar{\tau})^2}{\sum_{i=0}^{L-1} |a_i|^2} \right)^{\frac{1}{2}}$$

$$\sigma_{\tau} = 0.445 \mu\text{s}$$

Therefore,

$$\sigma_{\tau} \ll \sigma_{\tau}^{Max}$$

So, it is convenient to use RMS delay spread instead of Maximum Delay Spread.

VI. CHANNEL ESTIMATION FOR MIMO SYSTEM

MIMO is extremely key technology in 3G, 4G and 5G communication system but more specifically in 5G, massive MIMO is prime technology to increase the data rate over the wireless channel. In MIMO, multiple transmit and receive antennas are utilized in transmitter and receiver respectively to increase data rate possibly by transmitting multiple data stream in parallel, and in between each transmit and received antennas, there are multiple fading channel which is done by the key technology, called spatial multiplexing.

Let considering t-transmit antennas and r received antennas in transmitter and receiver. Therefore, the mathematical model for MIMO can be represented as,

$$\bar{y} = H\bar{x} + \bar{w}$$

Where, H is $r \times t$ channel matrix with r number of rows and t number of columns.

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{1t} \\ h_{12} & h_{22} & h_{2t} \\ h_{r1} & h_{r2} & h_{rt} \end{bmatrix}$$

For a 4×4 MIMO system,

The transmit vector = $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, and received vector = $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$

Therefore, the system model,

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{12} & h_{22} & h_{23} & h_{24} \\ h_{13} & h_{23} & h_{33} & h_{34} \\ h_{14} & h_{24} & h_{34} & h_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}$$

$$\text{Let, } H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 0 & -2 & 1 \\ 1 & 0 & 0 & -3 \end{bmatrix}$$

Then, the computation of U, S, and V gives,

$$U = \begin{bmatrix} -0.2887 & 0.4082 & -0.5000 & -0.7071 \\ -0.2887 & 0.4082 & -0.5000 & 0.7071 \\ -0.2887 & -0.8165 & -0.5000 & -0.0000 \\ 0.8660 & -0.0000 & -0.5000 & 0.0000 \end{bmatrix}$$

$$S = \begin{bmatrix} 3.4641 & 0 & 0 & 0 \\ 0 & 2.4495 & 0 & 0 \\ 0 & 0 & 2.0000 & 0 \\ 0 & 0 & 0 & 1.4142 \end{bmatrix}$$

$$V = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

where, $\sigma_1 = 3.4641, \sigma_2 = 2.4495, \sigma_3 = 2.0000, \sigma_4 = 1.4142$

Further,

$$\sigma_1 > \sigma_2 > \sigma_3 > \sigma_4$$

The channel capacity of a MIMO system with t-transmit and r-received antennas can increase by a factor of min (T,R) without additional transmit power or spectral bandwidth over the conventional SISO system for IID Rayleigh fading channel. The diversity technique improves transmission reliability whereas the spatial multiplexing maximizes the transmission rate as of MIMO channel capacity. The capacity of the random MIMO channel is analysed for three different cases: (i) IID Rayleigh Fading Channel, (ii) Separately correlated Rayleigh Fading Channel, and (iii) Keyhole Rayleigh Fading Channel. The ergodic capacity for IID fading MIMO channels are modelled to find the effect of antenna correlation on the MIMO channel capacity.

By using the SVD, the MIMO fading channel with the channel matrix H can be represented by decoupled parallel Gaussian sub-channels. Thus, the capacities of sub-channels add up, giving the overall instantaneous capacity for uniform or equal power allocation. The mean MIMO capacity for the Ergodic fading channel is

$$\langle C \rangle = E \left\{ W \log_2 \det \left(I + \frac{PQ}{t\sigma^2} \right) \right\}$$

Where, Q is the Wishart Matrix = $\begin{cases} HH^H, & r < t \\ H^H H, & r \geq t \end{cases}$

The capacity of the channel with T-transmit and R-received antennas under power constraint P equals,

$$\langle C \rangle = m(\log_2 e) E \left\{ \ln \left(1 + \frac{\gamma}{t} \lambda \right) \right\}$$

Using the basic assumptions, the Ergodic channel capacity can be written as,

$$C = \log_2 \det \left[I_{NR} + \left(\frac{Y}{N_t} \right) * H^H H \right]$$

Since using SVD, we can decompose a MIMO channel into RH parallel Gaussian channels, where, RH is the rank of the MIMO channel matrix. Here, we will find the capacity for the uniform and adaptive power allocation scheme. Uniform power allocation is employed when the channel state information (CSI) is available at the receiver but not at the transmitter. We can use adaptive power allocation based on Water-filling algorithm when CSI is available at the receiver as well as transmitter. Optimal power allocation for high and low SNR cases are discussed below.

A. Optimal Power Allocation Scheme

Power allocation plays a significant role in deciding the MIMO capacity. Power allocation was not an important issue in SISO, but for MIMO, it is one of the important parameters for increasing capacity. If we allot power equally to all transmit antennas or unequally to each transmit antenna, the capacity of the MIMO channel will be different. So, to allocate power adaptively we need the CSI at the transmitter, also since power allocation is done at the transmitter. Intuitively we will allocate more power to better channels than the worst. We may not allocate any power at all to some of the worst channels. In practical scenarios, we can allocate power near optimally for MIMO channels for two cases: high and low SNR regimes.

B. Uniform Power Allocation

The capacity indicates the best viable transmission data rate over the channel for miniscule probability of errors. Shannon provided the expression of the achievable communication rate of a channel with noise. If the transmission rate is greater than the capacity, the system is in outage and the receiver makes decoding errors with a non-negligible probability.

Let us derive the capacity of MIMO channels for uniform power allocation. Usually the channel state information is available at the receiver (CSIR) but not available at the transmitter (CSIT).

Assuming the input signal vector x .

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{NT} \end{bmatrix}, x_1, x_2, \dots, x_{NT} \text{ are mutually independent}$$

Assuming that the overall power of the transmitted signal is P and equal power is given to individual transmit antenna.

Then the covariance matrix of x ,

$$R_{xx} = \begin{bmatrix} \frac{P}{NT} & 0 & \dots & 0 \\ 0 & \frac{P}{NT} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{P}{NT} \end{bmatrix} = \frac{P}{NT} I_{NT} = \sigma^2 [I]$$

Using the singular value decomposition to find the singular values of the channel for,

$$\sigma_1 = 3.4641, \sigma_2 = 2.4495, \sigma_3 = 2.0000, \text{ and } \sigma_4 = 1.4142$$

So, that the singular values are

$$\sqrt{\lambda_1} = 3.46, \sqrt{\lambda_2} = 2.44, \sqrt{\lambda_3} = 2, \text{ and } \sqrt{\lambda_4} = 1.41$$

$$\text{or, } \lambda_1 = 12, \lambda_2 = 6, \lambda_3 = 4, \lambda_4 = 2$$

For optimal power distribution,

$$\gamma_i = \frac{P}{\sigma_i^2} \times \lambda_i$$

For, $L=4$ antenna elements, the SNR=16 dB for the average BER= 10^{-6} ,

$$\gamma_1 = 477.72, \gamma_2 = 238.86, \gamma_3 = 159.24, \gamma_4 = 79.62$$

Considering the power is distributed to the parallel channels, the power constraint becomes,

$$\sum_{i=0}^{L-1} \frac{1}{\gamma_0} - \frac{1}{\gamma_i} = 1$$

Therefore, solving the above equation becomes,

$$\frac{4}{\gamma_0} = 1 + \frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_3} + \frac{1}{\gamma_4} = 1.025$$

$$\gamma_0 = 3.90 \cong 4$$

Then, the capacity of the first channel becomes,

$$C = \log_2 \left(\frac{\gamma_1}{\gamma_0} \right) = 6.90 \text{ bits/sec/Hz}$$

The capacity of the second channel becomes,

$$C = \log_2 \left(\frac{\gamma_2}{\gamma_0} \right) = 5.90 \text{ bits/sec/Hz}$$

The capacity of the third channel becomes,

$$C = \log_2 \left(\frac{\gamma_3}{\gamma_0} \right) = 5.3150 \text{ bits/sec/Hz}$$

And the capacity of the fourth channel becomes,

$$C = \log_2 \left(\frac{\gamma_4}{\gamma_0} \right) = 4.3150 \text{ bits/sec/Hz}$$

So, the maximum capacity of the channel is 6.9 bits/sec/Hz.

CONCLUSION

This research article surveys with the beginning of the MIMO wireless communication with different fading scenario, channel estimation and application of Singular-Value-Decomposition (SVD) method to find out the channel capacity in a 4x4 MIMO antennas. It is seen that the channel capacity increases with the increment of antenna arrays.

REFERENCES

- [1] J. E. Brittain, "Electrical Engineering Hall of Fame: Harold H. Beverage," *Proc. IEEE*, vol. 96, pp. 1551–1554, Sep. 2008.
- [2] H. H. Beverage and H. O. Peterson, "Diversity receiving system of R.C.A. Communications, Inc., for radiotelegraphy," *Proc. IRE*, vol. 19, pp. 529–561, Apr. 1931.
- [3] H. O. Peterson, H. H. Beverage, and J. B. Moore, "Diversity telephone receiving system of R.C.A. Communications, Inc.," *Proc. IRE*, vol. 19, pp. 562–584, Apr. 1931.
- [4] F. A. Bartlett, "A dual diversity preselector," *QST*, vol. XXV, pp. 37–39, Apr. 1941.
- [5] D. G. Brennan, "Linear diversity combining techniques," *Proc. IRE*, vol. 47, pp. 1075–1102, Jun. 1959.
- [6] R. G. Vaughan and J. B. Andersen, "Antenna diversity in mobile communications," *IEEE Trans. Veh. Technol.*, vol. VT-36, pp. 147–172, Nov. 1987.
- [7] M. A. Jensen and Y. Rahmat-Samii, "Performance analysis of antennas for hand-held transceivers using FDTD," *IEEE Trans. Antennas Propag.*, vol. 42, pp. 1106–1113, Aug. 1994.
- [8] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Technical Journal*, pp. 41–59, Autumn 1996.
- [9] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, pp. 311–335, Mar. 1998.
- [10] G. G. Raleigh and J. M. Cioffi, "Spatio-temporal coding for wireless communication," *IEEE Trans. Commun.*, vol. 46, pp. 357–366, Mar. 1998.
- [11] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Selected Areas Commun.*, vol. 16, pp. 1451–1458, Oct. 1998.
- [12] M. A. Jensen and J. W. Wallace, "A review of antennas and propagation for MIMO wireless communications," *IEEE Trans. Antennas Propag.*, vol. 52, pp. 2810–2824, Nov. 2004.
- [13] Q. H. Spencer, B. D. Jeffs, M. A. Jensen, and A. L. Swindlehurst, "Modeling the statistical time and angle of arrival characteristics of an indoor multipath channel," *IEEE J. Selected Areas Commun.*, vol. 18, pp. 347–360, Mar. 2000.
- [14] J. W. Wallace and M. A. Jensen, "Modeling the indoor MIMO wireless channel," *IEEE Trans. Antennas Propag.*, vol. 50, pp. 591–599, May 2002.
- [15] M. L. Morris and M. A. Jensen, "Network model for MIMO systems with coupled antennas and noisy amplifiers," *IEEE Trans. Antennas Propag.*, vol. 53, pp. 545–552, Jan. 2005.
- [16] E. Larsson, O. Edfors, F. Tufvesson, and T. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Comm. Mag.*, vol. 52, pp. 186–195, Feb. 2014.